Periodic interactions between solitons and dispersive waves during the generation of noncoherent supercontinuum radiation

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Abstract: We present a numerical study of interactions between dispersive waves (DWs) and solitons during supercontinuum generation in photonic crystal fibers pumped with picosecond laser pulses. We show how the soliton-induced trapping potential evolves along the fiber and affects the dynamics of a DW-soliton pair. Individual frequency components of the DW periodically interact with the soliton resulting in stepwise frequency blue shifts. In contrast, the ensemble blue shifts of all frequency components in the DW appear to be quasi-continuous. The step size of frequency up-conversion and the temporal separation between subsequent soliton-DW interactions are governed by the potential well which confines the soliton-DW pair and which changes in time.

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1. Introduction

Supercontinuum (SC) radiation generated in optical fibers [1–3] provides light of broad bandwidth and extreme brightness enabling many applications in diverse areas of metrology [4], telecommunications [5], spectroscopy and microscopy. For many practical applications, affordable fiber and solid state lasers are becoming increasingly ubiquitous as convenient pump sources to generate SC in photonic crystal fibers (PCFs). These sources generate comparatively long pump pulses (ps or longer) leading to important differences in spectral and temporal properties of the resulting supercontinua compared to those generated with fs pulses.

Here we present a numerical analysis of long pulse, incoherent supercontinua concentrating on interactions of solitons and dispersive waves (DWs). The study was motivated by our increasing use of high power, incoherent SC light in the development of practical spectroscopy and microscopic imaging applications. These include the development of high sensitivity broadband absorption spectroscopy techniques [6-8] that are simpler to implement than corresponding techniques with low noise frequency combs generated by coherent supercontinua [9, 10]. Also we have developed fluorescence microscopy applications, capable of measuring the full optical state of the sample, including fluorescence spectrum, lifetime and polarization state [11,12]. For time correlated measurements, the temporal pulse behavior of incoherent supercontinua is an especially important characteristic. For the optimal deployment of such techniques a thorough understanding and quantification of the spectral and temporal properties of incoherent supercontinua is crucial: for example, significant intensity and spectral variations require longer averaging times in absorption measurements [13] or may affect time correlated measurements. At the heart of these phenomena are soliton-DW interactions. The physics of such interactions has been studied in detail in idealized situations, particularly for the case of individual soliton-DW pairs in the short pump-pulse regime [14–18]. Here, experiments and theory show excellent agreement with the theoretical behavior [19-21]. In this paper we present a numerical study of DW and soliton interactions occurring during incoherent SC generation. The expected sequence of events here is, briefly, as follows. First, a modulation instability generates symmetrical sidebands in the frequency domain due to four wave mixing (FWM) [2]. Raman induced frequency shifts split the input pulse into multiple fundamental solitons in the anomalous dispersion regime (ADR), which undergo soliton self frequency shift as they propagate along the fiber [22]. At the same time, a fraction of the pulse energy is transferred into the normal dispersion region (NDR) leading to the formation of dispersive waves. It is the interaction of solitons and DWs which dominates the next stage of SC generation. The modulation instability originates from noise and thus the solitons have, generally, different temporal widths, peak powers and center wavelengths creating an incoherent supercontinuum.

A particular focus is on the up-conversion process of frequency components present in the DW and their evolution in time within incoherent supercontinuum pulses. The study of individual frequency components of DWs and their intermittent interactions with a soliton leads to noteworthy phenomena which are not evident from the smooth ensemble behavior of the DW components. These phenomena are graphically illustrated and provide an intuitive understanding of processes shaping the spectral properties of long pulse SC. The results presented here are of importance to short wavelength generation in PCFs pumped with ps or ns lasers, e.g. fiber or microchip laser sources and its application for measurement.

2. Numerical Model

Our numerical simulation is based on the well-known generalized nonlinear Schrödinger equation (GNLSE) shown in Eq. (1) [18], which includes high-order dispersion and nonlinear terms:

$$\frac{\partial A}{\partial z} - \sum_{k\geq 0} \frac{i^{k+1}}{k!} \beta_k \frac{\partial^k A}{\partial t^k} = i\gamma (1 + i\tau_{shock} \frac{\partial}{\partial t}) \left(A(z,t) \int_{-\infty}^{+\infty} R(t') \left| A(z,t-t') \right|^2 dt' \right)$$
(1)

We defined input pulses to have 100 W peak power, at a center wavelength of 1064 nm and with 5 ps temporal profiles modeled to be of unchirped hyperbolic secant form. Details for other simulation parameters are given in [13]. In addition, we define the nonlinear coefficient as:

$$\gamma(\omega) = \frac{n_2(\omega)\omega}{cA_{eff}(\omega)}$$
(2)

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where n_2 is the nonlinear refractive index. For fused silica, the frequency variation of n_2 is typically small in the near-infrared regime [23] and thus is treated here as constant. A_{eff} is the effective mode area that leads to dispersion of the nonlinearity due to the frequency-dependence of A_{eff} [24]. Field components at longer wavelength experience a reduced nonlinearity due to the increased effective mode area, with a proportional influence on the interaction between solitons and dispersive waves. The result is that the trapped DW is confined by an ever weakening nonlinear effect. The fourth-order Runge-Kutta in the interaction picture method [25,26] is used to solve Eq. (1) numerically. The noise of the input field is modeled by adding one photon with random phase to each frequency bin of the pump pulse [27].

3. Results and discussion

3.1 Interaction between soliton and dispersive wave

During long pulse SC generation, a high order soliton splits into multiple fundamental solitons due to perturbations caused by Raman effects. The first ejected soliton features a shorter temporal width and higher peak power compared to solitons ejected later [28]. In what follows we focus on the dynamics of this first soliton and its corresponding DW. Other soliton-DW pairs undergo similar interactions, however they are often perturbed by collisions amongst each other. The first-ejected soliton-DW pair generally propagates in isolation and hence its dynamics are simpler to study. The simulated spectrograms in Fig. 1(a-c) show the dynamics of soliton-DW pairs at three different propagation lengths in the PCF. The three propagation lengths correspond to the situation before (29 m), during (36 m) and after (40 m) a prominent soliton-DW collision. The first soliton and the corresponding DW are indicated in Fig. 1. Figures 1(d-f) show the corresponding temporal intensity profiles of the soliton-DW pair. Figure 1(d) shows the situation before the soliton-DW interaction. The soliton decelerates due to Raman frequency shift in the ADR and changes the local refractive index due to SPM. The DW will thus catch up with the soliton and then experiences a nonlinear interaction with the soliton. During collision (Fig. 1(e)), the DW exchanges energy with the soliton by FWM [14,15,29–33]. The DW decelerates (Fig. 1(f)) and mostly remains trapped by the potential barrier presented by the decelerating soliton [14-17]. This soliton-DW interaction process continues: the soliton continues to decelerate, the DW catches up with the soliton again, interacts and rebounds. These periodic interactions lead to stepwise frequency blue-shifts every time the soliton and DW collide [15,31].



Fig. 1. Supercontinuum generation is shown by spectrograms at three different fiber lengths: (a) 29 m, (b) 36 m, (c) 40 m. The first soliton is indicated by an ellipse (dotted), and the corresponding DW by a rectangle, in the spectrograms and (d-f) in the time domain. The arc (dashed) illustrates how the DW first collides with the soliton and then gets reflected back. The phenomenon is also presented as an animation (Media 1).

3.2 From stepwise to quasi-continuous frequency shift

The periodic character of the DW-soliton interactions is evident in individual frequency components of the DW. Figure 2 shows the blue shift of the frequency component corresponding to the peak intensity of the DW as a function of propagation distance. The stepwise character of the frequency shifts is clearly seen. In order to quantify ensemble frequency changes of the soliton-DW interactions, we introduce the intensity-weighted average frequency of the DW as a function of propagation distance z:

$$\left\langle \omega_{DW}(z) \right\rangle = \frac{\int_{\omega_a}^{\omega_b} \omega \left| A_{DW}(\omega, z) \right|^2 d\omega}{\int_{\omega_a}^{\omega_b} \left| A_{DW}(\omega, z) \right|^2 d\omega}$$
(3)

where the limits of integration (ω_a and ω_b) are chosen to be -30 dB compared to the maximum intensity of the DW and $|A_{DW}|^2$ represents the spectral intensity of the DW. In contrast to the stepwise behavior of individual frequency components, this average frequency of the entire packet of trapped photons increases smoothly.

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Fig. 2. Weighted average frequency (red line) and frequency of the peak intensity component of the DW as a function of propagation distance in the fiber.

A closer inspection of the first ejected soliton and its corresponding DW shows how the collisions change the spectral characteristics of the DW. Figures 3(a-c) show magnified views of the DW spectrograms demarked with red boxes as in Fig. 1(a-c). During collision the frequency component corresponding to the maximum intensity changes in a discrete frequency step from ω_1 to ω_2 , as shown in Figs. 3(a-c) (c.f. Fig. 2). The trailing edge of the soliton creates a potential barrier by Kerr nonlinearity, which the DW cannot easily cross. Nevertheless it can be seen that some DW frequency components leak through the finite potential barrier (see Fig. 3(d)). These components are at the long-wavelength edge of the trapped DW, because of their velocity mismatch with respect to the soliton.

The stepwise frequency blue shifts experienced by components of the DW have been described as an optical analog of the so called "quantum bouncer" encountered in cold atom physics [14,31]. It is notable (see Fig. 2) that the size of subsequent frequency steps continuously decreases: after several collisions the DWs that are still trapped by a soliton undergo quasi-continous deceleration and frequency shift.

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Fig. 3. Magnified views of the first appearing DW: (a) before collision with the soliton, (b) DW in collision with the soliton, (c) after collision; (d) 3D plot of the trapped DW at 36 m (also presented as an animation in online Media 2). (e) Spectrogram at 100 m propagation distance. (f) DW in a weak trapping state at 100 m.

3.3 Evolving trapping potential

To gain insight into this phenomenon we study the evolution of the potential well generated by the soliton, which traps the DW. It consists of two parts [16,17]. The first is a local potential U_s created by the soliton SPM (Kerr nonlinearity). A further component U_L is induced by the decelerating frame of reference. The total potential is therefore:

$$U = U_s + U_L = 2\gamma P [1 + \frac{\tau_R}{T} \tanh(\frac{\tau}{T})] - \frac{4\tau_R \gamma^2 P_0}{15\beta_2} \tau$$
(4)

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where $P = P_0 [\operatorname{sech}(\tau/T)]^2$ is the soliton power, *T* is the soliton width and τ_R is the Raman response time. The time delay relative to the soliton is represented by τ . The potential well confines the DW near its minimum, trapping the DW behind the soliton. Because *U* depends on soliton parameters, such as wavelength and intensity, which change in time, the trapping potential evolves as the soliton-DW pair propagates along the fiber.

Figure 4 depicts how the soliton potential evolves: U_S decreases as the soliton peak power declines due to red-shift driven increases in mode area, temporal broadening, and energy loss. U_L also becomes shallower because inelastic Raman scattering reduces the energy of the soliton which in turn reduces the rate of self-frequency shift and results in a declining deceleration of the soliton [34]. The potential well thus becomes gradually shallower and the DW less tightly confined in the well. This leads to temporal dispersion of the DW which is clearly evident in Fig. 4(b) (also Fig. 2(f)): at 100 m the DW is twice the width it was at 29 m. The problem is an optical analog of a quantum mechanical wave packet moving in a changing potential well. As the well becomes continually shallower, collisions between DW and soliton become less energetic with reduced energy exchange.



Fig. 4. (a) Potential barrier U as a function of the fiber length between 20 m and 100 m fiber length. (b) Diagram of the DW trapped in the potential well at 29 m and 100 m fiber lengths.

4. Conclusion

We have examined soliton-DW interactions during the generation of incoherent supercontinuum radiation by numerical simulation of picosecond laser pulses propagating in a photonic crystal fiber. We used the weighted average of frequency components in the DW as well as the frequency component corresponding to the peak intensity in the DW to obtain insight into the frequency blue shift of the DW. We find that the frequency of the peak intensity component increases in steps due to interpulse four-wave mixing, whilst the ensemble average of frequency components transforms quasi-continuously. This is because the many interactions taking place between a soliton and the different DW components wash out the frequency shift behavior. Fiber dispersion and the wavelength dependence of the nonlinear properties result in shifting of the peak intensity frequency of the DW with decreasing step size and increasing bounce period. The group velocity mismatch between the soliton and DW decreases along the fiber and thus the FWM between the soliton and DW gradually transforms from a stepwise to a quasi-continuous process where the soliton trapping effect plays the main role in extending the short wavelength edge of the SC spectrum. The

#159070 - \$15.00 USD Received 1 Dec 2011; revised 17 Feb 2012; accepted 27 Feb 2012; published 5 Mar 2012 (C) 2012 OSA 12 March 2012 / Vol. 20, No. 6 / OPTICS EXPRESS 6323 soliton-induced potential, which keeps DWs from dispersing in time, decreases as the pulse propagates along the fiber resulting in increasing temporal dispersion of the DW.

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